Research Statement

Piotr Achinger

My current research interests lie at the intersection of algebraic geometry, homological algebra and commutative algebra. My research background is mainly in positive-characteristic algebraic geometry and homogeneous spaces, but this reflects the particular problems I was given, and I am eager to try to tackle some problems in nearby areas. For example, I am interested in the theory of abelian varieties, especially in prime characteristic, and recently also arithmetic geometry.

At Warsaw University, I worked with Prof. Adrian Langer and wrote my master's thesis concerning the Frobenius morphism on smooth quadric hypersurfaces under his supervision. During this work, I got interested in derived categories of coherent sheaves and positive-characteristic commutative algebra and had plenty of ideas for future study. I have recently moved to the Hausdorff Center for Mathematics in Bonn, where I work under supervision of Prof. Nicolas Perrin. Using Frobenius splitting methods and toric degeneration techniques, I prove results concerning singularities of subvarieties of some G-varieties.

I would like to work with Prof. Ogus on positive characteristic algebraic geometry or with Prof. Eisenbud on commutative algebra (especially maximal Cohen-Macaulay modules) or with Prof. Coleman or Prof. Ribet on the algebro-geometric side of arithmetic geometry (especially abelian varieties in positive characteristic).

My future advisor could assume, among other things, my familiarity with most of the content of the following books: *Algebraic Geometry* by R. Hartshorne, *Fourier-Mukai Transforms in Algebraic Geometry* by D. Huybrechts, *Abelian Varieties* by D. Mumford, *Frobenius Splittings Methods in Geometry and Representation Theory* by M. Brion and S. Kumar, and *Commutative Algebra with a View Toward Algebraic Geometry* by D. Eisenbud.

Master's thesis *Frobenius push-forwards on quadrics*

My master's thesis project *Frobenius push-forwards on quadrics* (`arXiv:1005.0594v1`, to appear in *Communications in Algebra*), written under supervision of Prof. Adrian Langer, addressed the following problem: *Let X be the smooth n-dimensional quadric hypersurface over an algebraically closed field of characteristic $p > 0$. Denote by F the Frobenius morphism on X and by F^s is s-th composition. Describe the sheaf $F^s_*\mathscr{L}$ for any line bundle $\mathscr{L}$ on X.* This was treated in a paper by Langer, but his description worked only for $s = 1$ and $p > 2$. In my work, I found a simpler and more transparent way to find this decompositions, which worked for arbitrary s and p and was more explicit than Langer's. This allowed me to extend the results of Langer on tilting bundles on X.

There are several questions which arose from this. For example, since the problem uses the description of maximal Cohen-Macaulay modules over the quadric in a substantial way, I got interested in the theory of Cohen-Macaulay modules and matrix factorizations. I heard a talk by Prof. Buchweitz on Orlov's theorem on triangulated categories of singularities and this connection, which can be intuitively predicted by looking at the case of quadrics, inspired me to investigate the action of the Frobenius morphism of these triangulated categories. Another interesting direction would be to extend these results to singular quadrics (I am convinced that my results would remain in a sense true in this case).

I am interested in finding a splitting criterion on hypersurfaces in positive characteristic, namely to answer the following question: *Let X be a Frobenius split hypersurface in $\mathbb{P}^n$ and $\mathcal{E}$ a vector bundle on X. If $F^*\mathcal{E}$ is ACM, does it follow that $\mathcal{E}$ is a direct sum of line bundles?*

Frobenius splittings

In Fall 2010, I moved to the Hausdorff Center for Mathematics in Bonn to work with Prof. Nicolas Perrin. My plan, which I have discussed with the faculty, was to join their PhD programme for a year to gain research experience and apply to more competitive places afterwards. The problem I was given was: *It is well known that Schubert varieties are normal and Cohen-Macaulay. Prove in a geometric way the same result for B-orbit closures in the product of two Grassmannians.* The idea was to use Frobenius splitting methods. At the moment of writing this research statement, I have proved normality in positive characteristic and I made significant progress towards lifting it to characteristic zero.

During this work, I got interested in toric degenerations of varieties appearing in representation theory. It seems that there is a deep connection between toric degenerations and Frobenius splittings. I plan to to apply the methods from the recent paper of Dave Anderson (which relies on previous work of Lazarsfeld and Mustaţă) to various types of Frobenius splittings. I believe that my idea of using toric degenerations could prove useful e.g. to determine *which* Schubert varieties are diagonally split.

My interest in toric varieties on one hand and in Frobenius splittings on the other led me to the discovery of a particularly nice and short proof of the result of Thomsen which computes the Frobenius direct images of line bundles on toric varieties. This work has been recently posted on the arXiv (`arXiv:1012.2021`).

Abelian varieties in positive characteristic

I am interested in Frobenius push-forwards and pull-backs of vector bundles on abelian varieties. I started working on my own on the extension of the results of Tango on the Frobenius push-forward of the structure sheaf on elliptic curves to higher-dimensional abelian varieties. For example, it is an easy fact that if X is an ordinary abelian variety then $F_*\mathscr{O}_X$ is the sum of all p-torsion line bundles, and it seems that in any case we have $F^*F_*\mathscr{O}_X = \mathscr{O}_X^p$. I plan to approach this problem via Cartier descent and the study of the induced action of the Frobenius kernel on $\mathscr{O}_X^p$.

In their recent work, Brenner and Kaid found bundles $\mathcal{E}$ such that $F^*\mathcal{E} = \mathcal{E}$ on Fermat curves. It could be interesting to study this phenomenon for higher-dimensional abelian varieties and its connection with the algebraic fundamental group scheme.

As I am interested in singularities in positive characteristic, I was happy to find out that the conjecture of Mustaţă and Smith on the equality of log-canonical threshold and F-pure threshold for infinitely many p would imply Serre's conjecture on ordinary reductions of abelian varieties. This connection between two areas I am interested in shows that my background could be useful also for researchers interested in the arithmetic of abelian varieties.

I always wanted to learn arithmetic geometry, but in Warsaw nobody does it; since I moved to Bonn, I have the opportunity to learn it from Prof. Faltings. However, I do not have any research experience in this field yet.

My research experiences described above make me confident that I have the skills and the perseverance necessary to become a good mathematician. I think that my main strength lies in the ability of asking good questions – when I check the state of research regarding some idea of mine, it often turns out that the question has been recently solved or posed by professional mathematicians. On one hand it makes me a little disappointed to be just a few years late, but on the other - it shows me that I am progressing in the right way. I believe that the University of California, Berkeley would be the perfect place to fully develop my potential.